Exact Z^2 scaling of pair production in the high-energy limit of heavy-ion collisions

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The two-center Dirac equation for an electron in the external electromagnetic field of two colliding heavy ions in the limit in which the ions are moving at the speed of light is exactly solved and nonperturbative amplitudes for free electron-positron pair production are obtained. We find the condition for the applicability of this solution for large but finite collision energy, and use it to explain recent experimental results. The observed scaling of positron yields as the square of the projectile and target charges is a result of an exact cancellation of a nonperturbative charge dependence and holds as well for large coupling. Other observables would be sensitive to nonperturbative phases.

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There is a very small number of problems in physics that yield to an exact solution. Remarkably, electromagnetic production of free electron-positron pairs in the high-energy limit of peripheral, heavy-ion collisions can be described by a two-center, time-dependent Dirac equation which can be solved exactly and in closed form [1]. In this Letter, we study this exact solution and discuss its implications for recent experiments performed at CERN's SPS [2], and possible future experiments at new facilities such as BNL's RHIC and CERN's LHC. (See Refs. [3,4] for relevant reviews of this field.)

Perturbative calculations have been held suspect at high energies and for the heaviest projectiles, e.g. because the coupling constant is not small ($Z\alpha \sim 0.6$) [2–6]. It is therefore surprising that positron yields observed from pair production in peripheral collisions of Pb^{82+} ions at 33-TeV on a Au target scale as $Z_T^2 Z_P^2$, and that the observed positron-momentum distributions display an overall good agreement with the leading-order perturbation-theory calculations [2]. The exact nonperturbative solution presented here explains these effects and is also consistent with the observed enhancement of the positron yields at small positron momentum. For future experiments, we indicate what observables would show complete agreement with second-order perturbation theory, and what other observables should be measured in order to detect nonperturbative effects.

The relativistic scattering problem of an electron in the external field of two point-like charges (ions), moving on parallel, straight-line trajectories in opposite directions at speeds which approach the speed of light, and at an impact parameter $2\vec{b}$, reduces in the high-energy limit to

$$
\begin{aligned}
i\frac{\partial}{\partial t}|\Psi(\vec{r},t)\rangle &= \left[\hat{H}_0 + \hat{W}_A(t) + \hat{W}_B(t)\right]|\Psi(\vec{r},t)\rangle, \\
\hat{H}_0 &\equiv -i\check{\alpha}\cdot\vec{\nabla} + \check{\gamma}^0, \\
\lim_{\gamma\to\infty}\hat{W}_A &= (\mathrm{I}_4 - \check{\alpha}_z)Z_A\alpha\delta(t-z)\ln\left[\frac{(\vec{r}_\perp - \vec{b})^2}{b^2}\right], \\
\lim_{\gamma\to\infty}\hat{W}_B &= (\mathrm{I}_4 + \check{\alpha}_z)Z_B\alpha\delta(t+z)\ln\left[\frac{(\vec{r}_\perp + \vec{b})^2}{b^2}\right], \quad (1)
\end{aligned}
$$

where $\gamma \equiv 1/\sqrt{1-\beta^2}$, and $\beta \equiv v/c$ is the speed of the charges, Z_A and Z_B. (γ in the collider frame is related to γ_T in the target frame by $\gamma_T = 2\gamma^2 - 1$.) Equation (1) is written in the collider frame, with natural units ($c = 1$, $m_e = 1$, and $\hbar = 1$), α is the fine-structure constant, and $\check{\alpha}$ and $\check{\gamma}^\mu$ are Dirac matrices in the Dirac representation. The δ-function form of the interaction is a high-energy limit of the exact interaction in a *short-range* representation for the electron's Dirac-spinor, chosen to remove the interaction at asymptotic times [7–9].

For Eq. (1) to apply, one assumes first that the ions are sufficiently energetic and massive so that the deviation from straight-line trajectories can be neglected [4]. Second, one assumes peripheral collisions without nuclear interaction. Purely electromagnetic events can be distinguished experimentally from nuclear events by observing the full-energy projectile ion after the collision in coincidence with the produced electrons or positrons [2]. One also assumes the ions are moving at the speed of light. Indeed, for the recent experiments at CERN [2], $\beta \approx 0.99$, and for future experiments possible at RHIC, $\beta \approx 0.9999$. Finally, to obtain the δ-function form of the interaction, one assumes that $\gamma \gg |\vec{r}_\perp \pm \vec{b}|$, and $2b$.

Equation (1) displays a unique electromagnetic interaction. The ion with charge Z_A is moving to the right at the speed of light. Its electromagnetic potential in the representation chosen here is Lorentz contracted to the plane transverse to its trajectory, (the light front $z = t$), hence the $\delta(z - t)$ functional dependence. Likewise, the ion with charge Z_B carries with it at the other light front, ($z = -t$), a plane of singular interaction moving to the left at the speed of light. Anywhere but on the light fronts, Eq. (1) reduces to the free Dirac equation. The Dirac plane waves $\{|\chi_p(\vec{r},t)\rangle = \exp(-iE_pt)\exp(i\vec{r}\cdot\vec{p})|u_p\rangle\}$ which satisfy the free Dirac equation are each characterized by the quantum numbers $p \equiv \{\vec{p},\lambda_p,s_p\}$; the momentum $\vec{p}$, the sign

of the energy $E_p = (-1)^{\lambda_p}\sqrt{p^2+1}$, and the spin $s_p = \pm$. Explicit forms for the four four-spinors $|u_p\rangle$ are given in Ref. [4], and $p_\pm \equiv p_z \pm E_p$.

The scattering problem for the electron is defined by Eq. (1) and by plane-wave asymptotic initial and final states. One solves for the transition amplitude $A_k^{(j)}$,

$$\lim_{t\to-\infty} |\psi^{(j)}(\vec{r},t)\rangle = |\chi_j(\vec{r},t)\rangle,$$
$$\lim_{t\to+\infty} |\psi^{(j)}(\vec{r},t)\rangle = \sum_k A_k^{(j)} |\chi_k(\vec{r},t)\rangle, \qquad (2)$$

where $\sum_k$ stands for integration over $\vec{k}$ and summation over λ_k and s_k. We have obtained an exact, closed-form integral representation for the scattering amplitude $A_k^{(j)}$ in the following manner [1]. First we have observed that, as the ions are approaching from infinity, no change occurs in the region of space between them ($|z| < |t|$) until $t = 0$ when the two singular interaction planes collide. However, as each ion sweeps through space, it interacts with the single plane wave it encounters, resulting in a superposition of plane waves after it passes. Each δ-function interaction induces a phase-shift discontinuity in the wavefunction across each light front [7,8,1]. A phase shift induced on a plane wave by the passage of a single ion is not sufficient to produce pairs, but as the two phase-shift planes collide at $t = 0$, they interfere, and pairs are produced as a result. After the collision ($t > 0$), as the ions move apart, the solution in the space between them ($|z| < t$) is a new superposition of plane waves which is determined by the nontrivial boundary condition at the light fronts. We have calculated the transition amplitudes $A_k^{(j)}$ by integrating the flux of the conserved transition four-current which flows into this region across the light fronts. Two terms contribute to the amplitude corresponding to the two time orderings of the interaction of the electronic wavefunction with the two ions.

The transition amplitudes, $A_k^{(j)}$, are represented in terms of the transverse-momentum transfer distribution induced by a single ion, $Q_Z^{\vec{b}}(\vec{\kappa})$, which contains all the dynamics of the ion-electron interaction. When $\lambda_k = 0$ and $\lambda_j = 1$, $A_k^{(j)}$ is an amplitude for a transition from the negative continuum to the positive continuum, i.e. an amplitude for pair production. We have found [1],

$$A_k^{(j)} = \frac{i}{\pi}\int d\vec{p}_\perp \left\{ \frac{\sigma_k^j(\vec{p}_\perp)\; Q_{Z_B}^{\vec{b}}(\vec{k}_\perp - \vec{p}_\perp)\; Q_{Z_A}^{\vec{b}}(\vec{j}_\perp - \vec{p}_\perp)}{p_\perp^2 + 1 - j_- k_+} - \frac{\sigma_j^{k\dagger}(\vec{p}_\perp)\; Q_{Z_A}^{\vec{b}}(\vec{p}_\perp - \vec{k}_\perp)\; Q_{Z_B}^{\vec{b}}(\vec{p}_\perp - \vec{j}_\perp)}{p_\perp^2 + 1 - j_+ k_-} \right\}. \qquad (3)$$

The spinor part is $\sigma_k^j(\vec{p}_\perp) \equiv (2\pi)^3 \langle u_k | (I_4 - \check{\alpha}_z)(\check{\alpha}\cdot\vec{p}_\perp + \check{\gamma}^0)(I_4 + \check{\alpha}_z)|u_j\rangle$ and the momentum-transfer distribution is given by the Fourier transform of the phase shift at the light front [1],

$$Q_Z^{\vec{b}}(\vec{\kappa}) \equiv \frac{1}{(2\pi)^2}\int d\vec{r}_\perp\, e^{i\vec{r}_\perp\cdot\vec{\kappa}} \left[\frac{(\vec{r}_\perp - \vec{b})^2}{b^2}\right]^{-i\alpha Z} = \frac{1}{2\pi}\frac{\exp(i\vec{\kappa}\cdot\vec{b})}{\kappa^2 (b\kappa)^{-i2\alpha Z}} \int_{\xi>0} d\xi\, J_0(\xi)\, \xi^{1-i2\alpha Z}, \qquad (4)$$

where Z is the charge of the corresponding ion and $\vec{\kappa}$ is the transverse-momentum transfer. The integral over $\xi \equiv \kappa|\vec{r}_\perp - \vec{b}|$ in Eq. (4) should be regularized so as to avoid unphysical contributions from large, transverse distances, i.e. from $\xi > \kappa\gamma$. We studied several regularization schemes (for $\vec{\kappa} \neq 0$) which all gave [10]

$$Q_Z^{\vec{b}}(\vec{\kappa}) \to \frac{-i\alpha Z}{\pi}\frac{\exp(i\vec{\kappa}\cdot\vec{b})}{\kappa^2}\left[\frac{\Gamma(-i\alpha Z)}{\Gamma(+i\alpha Z)}\left(\frac{b\kappa}{2}\right)^{+i2\alpha Z}\right]. \qquad (5)$$

The exact amplitudes in the infinite γ limit are obtained by substituting the result of Eq. (5) in Eq. (3),

$$\begin{aligned}\lim_{\gamma\to\infty} A_k^{(j)} = &\left[\left(\frac{b}{2}\right)^{+i2\alpha(Z_A+Z_B)} \frac{\Gamma(-i\alpha Z_A)}{\Gamma(+i\alpha Z_A)}\frac{\Gamma(-i\alpha Z_B)}{\Gamma(+i\alpha Z_B)}\right] \\ &\times \frac{i}{\pi^3}\,\alpha^2 Z_A Z_B \int d\vec{p}_\perp (\vec{p}_\perp - \vec{k}_\perp)^{-2}\,(\vec{p}_\perp - \vec{j}_\perp)^{-2} \\ &\times \left\{ \frac{\sigma_k^j(\vec{p}_\perp)}{p_\perp^2 + 1 - j_- k_+} e^{i\vec{b}\cdot(\vec{j}_\perp + \vec{k}_\perp - 2\vec{p}_\perp)} \right. \\ &\times \left[|\vec{p}_\perp - \vec{k}_\perp|^{i2\alpha Z_A} |\vec{p}_\perp - \vec{j}_\perp|^{i2\alpha Z_B}\right] \\ &- \frac{\sigma_j^{k\dagger}(\vec{p}_\perp)}{p_\perp^2 + 1 - j_+ k_-} e^{-i\vec{b}\cdot(\vec{j}_\perp + \vec{k}_\perp - 2\vec{p}_\perp)} \\ &\times \left.\left[|\vec{p}_\perp - \vec{k}_\perp|^{i2\alpha Z_B} |\vec{p}_\perp - \vec{j}_\perp|^{i2\alpha Z_A}\right]\right\}. \qquad (6)\end{aligned}$$

The branch-point singularities for the intermediate momentum $\vec{p}_\perp = \vec{k}_\perp$ or $\vec{j}_\perp$ are an artifact of using Eq. (5) for $\vec{\kappa} = 0$. (The integral over $\vec{p}_\perp$ in Eq. (3) has no singularities.) We continue the analysis assuming an appropriate regularization at these points.

Equation (6) is nonperturbative, and already includes the interaction to all orders in αZ for any value αZ may have. Yet, its form is very similar to the high-energy limit of results obtained from the two-photon exchange diagrams of second-order perturbation theory [5], (which in the following we simply call the perturbative result). For high-energy collisions, there is therefore no reason to calculate higher order diagrams. As $\gamma \to \infty$, the only corrections to second-order perturbation theory calculations for free-pair production, including both higher orders and nonperturbative effects, are the *phases* in the square brackets. For small values of αZ, these phases tend to 1 and the perturbative limit is reproduced [1]. What are the observable nonperturbative effects for finite charges? The phase in the square brackets outside the integral over $\vec{p}_\perp$ has no physical implications, but the phases in the integrands may substantially alter the

physical predictions. We find, for example, that the high-energy limit for $|A_k^{(j)}|^2$ differs, in general, from the perturbative result. On the other hand, using Eq. (6) to calculate the integrated observable $\int d(2\vec{b})|A_k^{(j)}|^2$, we get

$$\frac{4}{\pi^4}\,\alpha^4 Z_A^2 Z_B^2 \int d\vec{p}_\perp\;(\vec{p}_\perp-\vec{k}_\perp)^{-4}\;(\vec{p}_\perp-\vec{j}_\perp)^{-4} \tag{7}$$
$$\times\left\{\frac{\left|\sigma_k^j(\vec{p}_\perp)\right|^2}{(p_\perp^2+1-j_-k_+)^2}+\frac{\left|\sigma_k^j(\vec{p}_\perp)\right|^2}{(p_\perp^2+1-j_+k_-)^2}\right.$$
$$\left.-\;2Re\frac{\sigma_k^j(\vec{p}_\perp)\sigma_j^k(\vec{j}_\perp+\vec{k}_\perp-\vec{p}_\perp)}{(p_\perp^2+1-j_-k_+)((\vec{j}_\perp+\vec{k}_\perp-\vec{p}_\perp)^2+1-j_+k_-)}\right\},$$

which is identical to the perturbative result. The integration over the impact parameter results here in cancellation of the nonperturbative phases, as $\int d(2\vec{b})\exp(i2\vec{b}\cdot\vec{p}') = (2\pi)^2\delta(\vec{p}')$. We conclude that while some observables are sensitive to the nonperturbative phases, other observables are not, e.g. because these phases are averaged to one by an integration. In these cases, observed results would agree with the second-order perturbation theory calculations, regardless of the size of αZ.

Would our results apply in an actual experiment, where γ is finite? Equation (1) is *incorrect* for large $r_\perp$ (or large b) where it describes an interaction which continually increases in strength. Implicit in using Eq. (1) for large, finite γ is a nontrivial assumption that large, transverse distances do not contribute to pair production. In the recent experiments at CERN [2], $\gamma \approx 10$, while in possible future experiments at RHIC and LHC, $\gamma \approx 100$ and $\gamma \approx 3000$, respectively. For these values of γ, Eq. (1) is respectively limited to pairs produced at transverse distances much smaller than 10, 100, and 3000 Compton wavelengths away from the ions. This restriction is consistent with an experimental observation according to which the average length scale for pair production in relativistic heavy-ion collisions is one Compton wavelength [2]. Yet, being concerned here not with averages but with complete distributions, we can not exclude the possibility of some pairs being produced at large, transverse distances from the highly charged ions, as long as the transverse-momentum transferred is sufficiently small. For Eq. (5) to be meaningful, the regulated integral of Eq. (4) must converge to the expression of Eq. (5) for ξ such that $|\vec{r}_\perp - \vec{b}| \ll \gamma$. The case of small coupling was previously studied [1]. The case of large αZ can be considered by the method of stationary phase. Expansion of Eq. (4) around the stationary point $\vec{r}_\perp - \vec{b} = 2\alpha Z\vec{\kappa}/\kappa^2$ confirms Eq. (5) for this case. The procedure is consistent if the stationary point is located at small distances from the ion, i.e. if and only if

$$|\vec{\kappa}| \gg \frac{2\alpha Z}{\gamma}\;. \tag{8}$$

It is interesting to find that Eq. (8) is trivially satisfied in two very different limits: in the perturbative limit of $\alpha Z \to 0$ and in the high-energy limit of $\gamma \to \infty$.

Thus, the results which we have first obtained for infinite γ, apply for finite γ as well. The only restriction is of Eq. (8), i.e. that the transverse-momentum transfer is not too small. For pair production, it is a sufficient condition to assume that either the initial or final (i.e. positron or electron) transverse-momenta are much larger than $2\alpha Z/\gamma$ where Z is the largest *free* charge involved in the collision. The argument goes as follows. There are three two-dimensional integration variables in Eq. (3). We first integrate over $\vec{p}_\perp$ to obtain simple combinations of the Bessel functions of the third kind, K_0 and K_1. We then use the condition that *one* of the two transverse momenta, $\vec{j}_\perp$ *or* $\vec{k}_\perp$, is much larger than $2\alpha Z/\gamma$ to apply a stationary phase calculation to one of the coordinate integrations. If, on the other hand, one of the charges is screened (a target charge, for example) the integral with it converges and there is no need to restrict the momentum conjugate to it. The last integral, over the other coordinate-integration variable, converges due to the Bessel functions which drop exponentially for large values of their arguments. Having thus proved that contributions for the 6-fold integral of Eq. (3) from large, transversal coordinates can be neglected, we can make the substitution of Eq. (5) and obtain Eq. (6). We remark that the convergence of the $\vec{p}_\perp$ integration to the Bessel functions occurs only for pair-production amplitudes for which $1 - j_\pm k_\mp > 0$, and is directly related to the mass gap between the two continua. It should be reconsidered for transitions within the same continuum.

We now consider the application of our results to the discussion of recent, pioneering experiments on pair production performed at CERN's SPS [2]. These experiments measured momentum spectra of positrons emitted from pair production in peripheral collisions of 33-TeV Pb^{82+} ions ($\gamma_T = 168$) and 6.4 TeV S^{16+} ions ($\gamma_T = 212$) with various targets (i.e. $(CH_2)_x$, Al, Pd, and Au). The charge dependence of the positron yield was reported with excellent precision. The target-charge dependence for the sulfur projectile is $Z_T^{1.99\pm0.02}$, and for the lead projectile is $Z_T^{2.03\pm0.03}$; both within $\sim 1\%$ agreement with the prediction of perturbation theory. The projectile-charge dependence was observed to be $Z_P^{2.0\pm0.1}$, also in very good agreement with perturbation theory. The positron momentum distributions for sulfur and lead projectiles are compared by scaling each spectrum by Z_P^2, and by scaling the sulfur data from $\gamma_T = 212$ to $\gamma_T = 168$ as $\ln^3(\gamma_T)$, as predicted by perturbation theory. The scaled distributions are observed to be approximately the same, and to agree reasonable well with two-photon perturbation theory (see discussion in [2]), except for enhancements for the lead projectile at very low (< 2 MeV/c) and high (between 8 MeV/c and 12 MeV/c) momentum. The authors of Ref. [2] note that the variation of the scaled momen-

tum distribution with the projectile charge, and not the target charge, is unexplained.

The observed $Z_T^2 Z_P^2$ charge dependence of the single-positron yields, even for very large charges, is consistent with the charge dependence we have obtained for the nonperturbative, high-energy limit (see Eq. (7)). It agrees with perturbation theory but is not a perturbative effect. Nonperturbative phases in the exact amplitudes make them different from second-order perturbation theory results, but these phases cancel for calculations of total cross sections. Our theoretical prediction of a Z^2 dependence of the total cross section in the high-energy limit implies that multiple-pair production in very high-energy collisions cannot be inferred from a measurement of the charge dependence of the total positron yield [2]. We suggest that the two regions of excess cross section observed in the experiment have a common origin: an enhancement over perturbation theory for small values of the transverse-momentum transfer, for which Eq. (4) diverges. We found that agreement with the perturbative result is restricted by Eq. (8). Assuming that in the collider frame pairs are produced isotropically, this restriction, formulated for the transverse momentum, may translate to a restriction on the total positron momentum: $j_\perp \sim j_z \sim j >> 2Z_P\alpha/\gamma$. Taking $>>$ to be a factor of 10, we then predict, in very good agreement with the observed scaled spectra, that the perturbative result for the positron momentum distributions is valid for $j > 0.4$ MeV/c for the sulfur data, and $j > 2$ MeV/c for the lead data. The excess cross section observed at high momentum for the lead projectile is consistent with an enhancement in the cross section at low transverse momentum in the collider frame after a relativistic transformation to the target frame is applied. The absence of an observed target-charge dependence for the scaled distributions, is most likely attributable to screening by the atomic electrons [11].

In conclusion, we have shown that the exact, nonperturbative solution and the two-photon exchange diagrams of second-order perturbation theory give exactly the same results for free-pair production yields integrated over the impact parameter, as long as the transverse-momenta transferred from the ions to the electron are larger than $2\alpha Z/\gamma$. The leading-order perturbative calculations *for this observable* are therefore exact not only at the perturbative limit of $\alpha Z \ll 1$ but also in the high-energy limit of $\gamma \gg 1$. This explains recent experimental results according to which production rates scale as $Z_P^2 Z_T^2$, even for large charges. New nonperturbative effects could be detected by measuring observables different from the integrated, inclusive production rate that was measured in these experiments. The exact amplitudes of Eq. (6) include nonperturbative phases which may have an observable effect, e.g. if one does not integrate over the impact parameter $2\vec{b}$. We expect these phases to strongly influence the theoretical predictions for correlations and multiple-pair production. Several issues deserve further study. These include pair production at large, transverse distances from the ions and bound-free production for which one should obtain the solution *on* the light fronts themselves. For other calculations, Eq. (6) as well as the physical picture that led to it, are likely to become useful theoretical tools [12].

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[11] Screening begins at the K-shell radius (a_K) and is assumed complete at the Thomas-Fermi radius (a_{TF}), (see also [14]). As either a_K or a_{TF} may be larger than γ, more work is required for detailed comparison. However, since the divergence in Eq. (4) is slow (logarithmic) and the charge dependence of $a_{TF} \sim 1/(Z_T^{1/3}\alpha)$ is weak, the target-charge dependence of the scaled momentum distributions for fixed-target experiments will be very weak.

[12] After we completed this work, two manuscripts have appeared in the Los-Alamos preprint archive discussing the high-energy limit of pair-production amplitudes in heavy-ion collisions [15,16]. Although they do not make the connection to recent experiments that we make here and do not discuss the essential condition of Eq. (8), these manuscripts reproduce and confirm various parts of our results and offer additional insight and perspective.

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